# STABILITY OF IDEAL INCOMPRESSIBLE FLOW WITH CONSTANT VORTICITY IN AN ELLIPTIC CYLINDER

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A large number of papers are devoted to the study of the stability of rotating flows. Significant progress has been made in understanding the particular idealized case, viz., the stability of ideal fluid flows with circular streamlines (see, e.g., [1-7]). A study of this problem made it possible to establish the existence of two fundamental mechanisms of instability in rotating flows, viz, "centrifugal" and "shear." An understanding of these instability mechanisms became the basis for modeling a whole series of transition and turbulence phenomena [2, 4, 7-12]). At the same time actual flows are frequently only approximately circular. Hence the question arises as to the influence of small deviations from circular geometry on the flow stability. This problem was studied in [13-16]. In [13-14] experimental and theoretical investigations were carried out on the stability of rotating fluid inside an elliptic cylinder with a small eccentricity. Instability leading to inflections of the axis of rotation was experimentally observed. In order to theoretically study this instability, a model based on Galerkin method using two well-chosen base functions was suggested. The existence of instability in such a formulation appears, at the first glance, surprising since it is known that rigid body rotation has a large stability margin [4]. The problem of the stability of linear vortex in a potential flow which is qualitatively close to [13-14] was investigated in [15-16]. The vortex core was assumed to be subject to small deformation so that the shape of its cross-section is close to an ellipse with a small eccentricity. Computations using small disturbance theory also showed the existence of instability associated with the inflexion of the axis of rotation. As in [13-14] and also in [15-16], theoretical investigation is limited to the study of flow stability with respect to disturbances of a particular type within the framework of linear theory. The stability of flow [13-14] inside an elliptic cylinder is the simplest among a large class of problems associated with stability of deformed fluid rotation and deserves detailed study. In the present paper results are given for the stability of this flow with respect to the general form of disturbances. Small disturbance theory is used in terms of the small parameter $\varepsilon$. According to computations, the flow is always unstable, even to the first-order approximation in $\varepsilon$, with respect to three-dimensional disturbances with wavelength $2\pi/k$ along the axis of rotation. The corresponding wave numbers k continuously fill even number of segments of width of the order $\varepsilon$. At the center of each segment (points $k_0$) the growth rate of disturbances is a maximum. The values of $k_0$ correspond to conditions for first order singularity of the problem. As regards the physical aspect of instability, we note that its mechanism is similar to the known "resonant interactions" [17] in the complex case when the plane waves are not the solutions to the linear problem. The result obtained can be interpreted as a variation of Hasselman's [18] statement on the instability of finite amplitude wave in the presence of corresponding "resonance triad." Here, resonance conditions take the form of already mentioned singularity conditions. The deformation of the rotational flow by the elliptic walls plays the role of the "first" wave splitting into two "resonant" waves. The Galerkin method is not used in this paper but the complete linearized equations of motion are solved. Hence the results presented here generalize [13-14] for the type of disturbances considered in these studies. There is a qualitative agreement here with the conclusions of [13-14].

1. Let us formulate the problem. Consider an elliptic cylinder whose surface is defined by the following equation in cylindrical coordinate system (r, θ, z)

$$r = F(\theta) \equiv (1 - \varepsilon \cos 2\theta)^{-1/2}. \tag{1.1}$$

If $a$ and b ($a$ > b) are the semi-axes of the ellipse lying in the normal plane of the cylinder, then $\varepsilon \equiv (a^2 - b^2)/(a^2 + b^2)$. The interior of the cylinder is filled by an ideal, incompressible fluid whose velocity field (U, V, 0) is

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$$U = -\varepsilon r \sin 2\theta,\ V = r(1 - \varepsilon \cos 2\theta). \tag{1.2}$$

In the corresponding vorticity field the only nonzero quantity is the z-component and it is a constant. All parameters are reduced to nondimensional form using the reference length $R \equiv ab\sqrt{2/(a^2+b^2)}$ and reference velocity $\Omega R$. The quantity $2\Omega$ is defined as the vorticity of the basic flow. The velocity field (1.2) is an exact solution of the equations of motion and satisfies the condition of impermeability on (1.1).

Consider infinitely small three-dimensional disturbances to the flow (1.2). Linearized equations of motion for the velocity field (1.2) are:

$$\begin{gathered} Lu + \frac{\partial U}{\partial r} u + \frac{1}{r}\frac{\partial U}{\partial \theta} v - \frac{2}{r} V v = -\frac{\partial p}{\partial r}, \\ Lv + \frac{\partial V}{\partial r} u + \frac{1}{r}\frac{\partial V}{\partial \theta} v + \frac{1}{r}(Uv + Vu) = -\frac{1}{r}\frac{\partial p}{\partial \theta}, \\ Lw = -\frac{\partial p}{\partial z},\ \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0. \end{gathered} \tag{1.3}$$

u, v, w, and p denote disturbance fields of radial, peripheral, axial velocity components, and the pressure disturbance; $L \equiv \partial/\partial t + U\partial/\partial r + (1/r)V\partial/\partial\theta$. Boundary conditions for (1.3) consist in equating to zero the normal velocity component on (1.1)

$$Fu - \frac{dF}{d\theta} v = 0 \quad \text{for} \quad r = F(\theta) \tag{1.4}$$

and boundedness of solutions at r = 0. In view of the invariance of equations and boundary conditions relative to displacements in z and t, the solution to the problem is sought in the form

$$(p,\ u,\ v,\ w) = (p_a,\ u_a,\ v_a,\ w_a)e^{i(kz-\omega t)}. \tag{1.5}$$

Amplitudes $p_a$, $u_a$, $v_a$, and $w_a$ are functions only of coordinates r and θ. Substituting (1.5) in (1.3) and (1.4) we get the eigenvalue problem in ω and corresponding eigenfunctions. If there exists at least one disturbance with Im ω > 0, then the flow is unstable.

2. The problem of determining the eigenvalues ω in the present formulation is very difficult and will be solved iteratively with respect to the small parameter ε. Computations will be made for the zeroth and first-order approximations.

Assuming the solution to be analytical functions of ε, we write them and F(θ) for small ε in a series (ν = 0, 1, 2,...):

$$(p_a,\ u_a,\ v_a,\ w_a,\ \omega,\ F) = \sum_{\nu=0}^{\infty} \varepsilon^{\nu}(p_\nu,\ u_\nu,\ v_\nu,\ w_\nu,\ \omega_\nu,\ f_\nu). \tag{2.1}$$

Besides, put $k = k_0 + \varepsilon k_1$. The last expression is used because discrete sum of fixed values of $k_0$ will correspond to cases of instability. The quantity $k_1$ makes it possible to evaluate k in the neighborhood of $k_0$. Substituting (2.1) in (1.3) and (1.4) and equating terms of the same order of ε, we get

$$\begin{gathered} L_0 u_\nu - 2v_\nu + \frac{\partial p_\nu}{\partial r} = G_{1\nu}, \qquad L_0 v_\nu + 2u_\nu + \frac{1}{r}\frac{\partial p_\nu}{\partial \theta} = G_{2\nu}, \\ L_0 w_\nu + ik_0 p_\nu = G_{3\nu},\ \frac{\partial u_\nu}{\partial r} + \frac{u_\nu}{r} + \frac{1}{r}\frac{\partial v_\nu}{\partial \theta} + ik_0 w_\nu = G_{4\nu}, \end{gathered} \tag{2.2}$$

where $L_0 \equiv -i\omega_0 + \partial/\partial\theta$. When ν = 0 for all $l$ ($l$ = 1, 2, 3, 4) the right-hand side $G_{l0} = 0$. In equations for the first-order approximation (ν = 1), functions $G_{l1}$ contain linear functions of the zeroth-order approximation and the quantity $\omega_1$. For example,

$$G_{11} \equiv i\omega_1 u_0 + \left(r\frac{\partial u_0}{\partial r} + u_0\right)\sin 2\theta + \frac{\partial u_0}{\partial \theta}\cos 2\theta.$$

Boundary conditions (1.4) for the zeroth- and first-order approximations (ν = 0; 1) give

$$u_0(1,\theta) = 0, \quad u_1(1,\theta) = \frac{df_1(\theta)}{d\theta} v_0(1,\theta) - f_1(\theta)\frac{\partial u_0}{\partial r}(1,\theta). \tag{2.3}$$

These conditions are "removed" in the usual way from the actual boundary (1.1) to the neighboring circle r = 1. The second boundary condition relates to the boundedness of solutions for any approximation at r = 0.

TABLE 1

| Intersection points \ n | 0 | 1 | 2 | 3 | Intersection points \ n | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|
| (1; 1) | 1,579<br>0 | 2,326<br>0,038 | 3,035<br>0,044 | 3,731<br>0,042 | (1; 3) | 2,630<br>—0,435 | 3,614<br>—0,331 | 4,468<br>—0,274 | 5,266<br>—0,238 |
| (2; 2) | 3,286<br>0 | 4,125<br>0,014 | 4,916<br>0,019 | 5,679<br>0,020 | (3; 1) | 2,630<br>0,435 | 3,556<br>0,361 | 4,390<br>0,314 | 5,182<br>0,281 |
| (3; 3) | 5,061<br>0 | 5,928<br>0,007 | 6,753<br>0,010 | 7,551<br>0,012 | (2; 3) | 4,046<br>—0,165 | 4,929<br>—0,128 | 5,754<br>—0,106 | 6,546<br>—0,091 |
| (1; 2) | 2,203<br>—0,292 | 3,075<br>—0,197 | 3,858<br>—0,154 | 4,605<br>—0,129 | (3; 2) | 4,046<br>0,165 | 4,912<br>0,147 | 5,729<br>0,134 | 6,516<br>0,123 |
| (2; 1) | 2,203<br>0,292 | 3,034<br>0,241 | 3,805<br>0,210 | 4,550<br>0,186 | | | | | |

3. Consider the problem (2.2) and (2.3) for the zeroth order. Its solutions are the widely known inertial waves in rigid body rotation of fluid [4, 19] for the particular case of the circular cylinder. For harmonics proportional to $\exp(im\theta)$, (2.2) and (2.3) give

$$p_0 = \alpha J_m(\eta_m r)e^{im\theta}. \tag{3.1}$$

Here $\alpha$ is an arbitrary complex constant; $J_m$ is a Bessel function of order m; $\eta_m^2 \equiv k_0^2\Delta_m/\sigma_m^2$; $\sigma_m \equiv m - \omega_0$; $\Delta_m \equiv 4 - \sigma_m^2$. The quantities $\omega_0$ and $k_0$ are related by the dispersion equation

$$\sigma_m\eta_m dJ_m(\eta_m)/d\eta_m + 2mJ_m(\eta_m) = 0. \tag{3.2}$$

It is known [4, 19] that the spectrum of $\omega_0$ is purely real and $m - 2 < \omega_0 < m + 2$.

4. For the first-order problem (2.2) and (2.3) the form of solutions and the quantity $\omega_1$ can be obtained by simple but tedious computations. They reduce to the determination of correction to the inertial waves (3.1) caused by differences in geometry from the circular. The most complex element in these computations is the solution of nonhomogeneous equations (2.2). A similar technique was used for another problem, e.g., in [15]. We observe that analytical calculations are significantly simplified by using equations for pressure disturbances obtained from (1.3) in [13]. Without pausing on computations we present the results.

The basic result is formulated in the form of certain conclusions. For the zeroth-order approximation in harmonics (3.1) with arbitrary m, $\omega_1$ is always real. This corresponds to stability to the first order. Instability occurs only in the case of singularity when the zeroth-order disturbance characterized by frequency $\omega_0$ and wavenumber $k_0$ is given by the superposition of two modes with different m ($m_1$ and $m_2$) such that $m_1 - m_2 = \pm 2$. However, such disturbances could be either stable or unstable. In the latter case, the range of wavenumbers for instability is $|k_1| < k_{max}$. Here, the amplification rate is a maximum at $k = k_0$ and monotonically decreases away from the limits of the interval, becoming zero when $k = k_0 \pm \varepsilon k_{max}$. The most dangerous disturbances are those whose modes $m_1$ and $m_2$ have identical number of zeros of radial velocity component inside the flow (when $0 < r < 1$). All such disturbances are unstable and their amplification rates are almost identical.

Let us clarify these conclusions. The "dangerous" disturbances in the zeroth-order approximation are:

$$p_0(r, \theta) = \alpha J_{n+1}(\eta r)e^{i(n+1)\theta} + \bar{\alpha} J_{n-1}(\bar{\eta} r)e^{i(n-1)\theta}. \tag{4.1}$$

Here $\alpha$ and $\bar{\alpha}$ are arbitrary complex constants; $\eta \equiv \eta_{n+1}$; $\bar{\eta} \equiv \eta_{n-1}$; n is an arbitrary integer. A bar over symbols denotes independent variables and not complex conjugates. It is convenient because of the symmetry of equations. The dispersion relation for harmonics in (4.1) are written in the form

$$\sigma\eta J_n(\eta) - (n+1)\bar{\sigma} J_{n+1}(\eta) = 0, \qquad \bar{\sigma}\bar{\eta} J_n(\bar{\eta}) - (n-1)\sigma J_{n-1}(\bar{\eta}) = 0, \tag{4.2}$$

where $\sigma \equiv \sigma_{n+1}$; $\bar{\sigma} \equiv \sigma_{n-1}$. In view of the singularity, the intersection points of the curves (4.2) are considered in the $k_0$-, $\omega_0$-plane. The intersecting family of curves (4.2) are

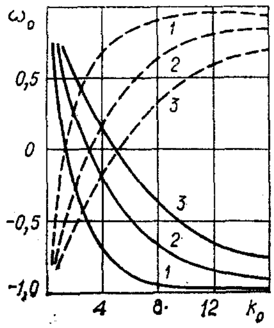


Fig. 1

concentrated in the band $n - 1 < \omega_0 < n + 1$. The shape of the families is shown in the figure for the case $n = 0$. Solid lines indicate $(n+1)$ harmonic, dashed lines denote $(n - 1)$ harmonic. Only the first three from the computed set of curves for each harmonic are shown in the figure. The numbers on the curves indicate the number of zeros of the function $u_0(r)$ on $0 < r \leq 1$. Each intersection point is denoted by a pair of integers $(q; s)$ corresponding to the numbers of the intersecting curves. Points with $q = s$ are called the principal intersection points, with $q \neq s$ being the secondary intersection points. When $n = 0$ (see Fig. 1), the curves are symmetrical about the axis $\omega_0$ and the principal intersection points are located on this axis. When $n \neq 0$, the picture of the branches of dispersion curves located within the band $-1 < \omega_0 - n < 1$ are qualitatively similar to that shown in the figure if the ordinate $\omega_0$ is replaced by $\omega_0 - n$. However, there is no symmetry of the curves about the axis $\omega_0 = n$ and the principal intersection points do not lie on this axis but they are close to it. The summary coordinates of the intersection points are given in Table 1 for n = 0, 1, 2, and 3 where the upper number in each box is $k_0$ and the lower number is $(\omega_0 - n)$.

Computations of $\omega_1$ at singularities $(\omega_0, k_0)$ give

$$\omega_1 = -\frac{1}{2} k_0 k_1 (c + \bar{c}) \pm \left\{\omega_{\max}^2 + \left[\frac{1}{2} k_0 k_1 (c - \bar{c})\right]^2\right\}^{1/2}, \tag{4.3}$$

where $c \equiv g/f$, $\bar{c} \equiv \bar{g}/\bar{f}$; $\omega_{\max}^2 \equiv h\bar{h}/f\bar{f}$; $f$, $\bar{f}$, $g$, $\bar{g}$, $h$, $\bar{h}$ are various real functions of $\omega_0$, $k_0$, and $n$:

$$f \equiv \left\{-\frac{4}{\sigma^2}[k^2 + (n+1)^2] + \frac{2}{\sigma}(n+1)\right\} J_{n+1},$$

$$\bar{f} \equiv \left\{-\frac{4}{\bar{\sigma}^2}[k^2 + (n-1)^2] + \frac{2}{\bar{\sigma}}(n-1)\right\} \bar{J}_{n-1},$$

$$h \equiv \frac{\pi}{2} l\eta\bar{S}_n[-\bar{\sigma}\bar{\eta}\bar{N}_n + \sigma(n-1)\bar{N}_{n-1}] + \frac{\bar{\sigma}}{4} J_{n+1} H,$$

$$\bar{h} \equiv -\frac{\pi}{2} l\bar{\eta} S_n[-\sigma\eta N_n + \bar{\sigma}(n+1) N_{n+1}] - \frac{\sigma}{4}\bar{J}_{n-1}\bar{H},$$

$$H \equiv -\eta^2 + \frac{1}{\sigma}[\mu\bar{\sigma} - \bar{\mu}(n+1)] + \frac{n(n+1)}{\Delta}[\bar{\Delta} - 2(\sigma + \bar{\sigma})] - \frac{\mu\bar{\sigma}n}{\sigma(\sigma^2 - \bar{\sigma}^2)}\left[\sigma^2 + \bar{\sigma}^2 - 2(n^2 - 1)\frac{\sigma\bar{\sigma}}{\eta^2}\right],$$

$$\bar{H} \equiv -\bar{\eta}^2 + \frac{1}{\bar{\sigma}}[\bar{\mu}\sigma - \mu(n-1)] - \frac{n(n-1)}{\bar{\Delta}}[\Delta + 2(\sigma + \bar{\sigma})] - \frac{\bar{\mu}\sigma n}{\bar{\sigma}(\sigma^2 - \bar{\sigma}^2)}\left[\sigma^2 + \bar{\sigma}^2 - 2(n^2 - 1)\frac{\sigma\bar{\sigma}}{\bar{\eta}^2}\right],$$

$$S_n \equiv \int_0^r J_{n+1}(\eta t) J_n(\bar{\eta} t)\, t^2 dt, \quad \bar{S}_n \equiv \int_0^r J_{n-1}(\bar{\eta} t) J_n(\eta t)\, t^2 dt,$$

$$g \equiv -\frac{\Delta}{\sigma}\left[1 + \left(\frac{n+1}{k_0}\right)^2\right] J_{n+1}, \quad \bar{g} \equiv -\frac{\bar{\Delta}}{\bar{\sigma}}\left[1 + \left(\frac{n-1}{k_0}\right)^2\right]\bar{J}_{n-1}. \tag{4.4}$$

The notations used are: $\sigma \equiv \sigma_{n+1}$; $\bar{\sigma} \equiv \sigma_{n-1}$; $\bar{\Delta} \equiv \Delta_{n-1}$; $\Delta \equiv \Delta_{n+1}$; $\eta \equiv \eta_{n+1}$; $\bar{\eta} \equiv \eta_{n-1}$; $\mu \equiv \sigma + 2$; $\bar{\mu} \equiv \bar{\sigma} - 2$; $l \equiv 4k_0^2(\omega_0 - n)/\sigma^2\bar{\sigma}^2$. Bessel functions $J_m$ and Neumann function $N_m$ are given by $J_m \equiv J_m(\eta)$, $\bar{J}_m \equiv J_m(\bar{\eta})$; $N_m \equiv N_m(\eta)$, $\bar{N}_m \equiv N_m(\bar{\eta})$.

It is seen from (4.3) that instability ($\operatorname{Im}\omega_1 > 0$) can occur if

$$\omega_{\max}^2 < -\left[\frac{1}{2} k_0 k_1 (c - \bar{c})\right]^2.$$

TABLE 2

| Intersection points \ $n$ | 0 | 0 | 1 | 2 | 3 | Intersection points \ $n$ | 0 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (1; 1) | 0,531 | 0,958 | 0,531 | 0,522 | 0,519 | (1; 3) | 0,006 | 0,024 | 0,028 | 0,040 | 0,048 |
| (2; 2) | 0,554 | 2,325 | 0,551 | 0,543 | 0,536 | (3; 1) | 0,006 | 0,024 | 0,015 | 0,011$i$ | 0,035 |
| (3; 3) | 0,559 | 3,701 | 0,557 | 0,551 | 0,546 | (2; 3) | 0,005 | 0,025 | 0,045 | 0,074 | 0,094 |
| (1; 2) | 0,012 | 0,035 | 0,065 | 0,102 | 0,121 | (3; 2) | 0,005 | 0,025 | 0,029 | 0,053 | 0,066 |
| (2; 1) | 0,012 | 0,035 | 0,039 | 0,052 | 0,075 | | | | | | |

The largest amplification rate $\omega_1 = \omega_{max}$ is achieved at $k_1 = 0$, where a whole range of wave numbers are unstable:

$$|k_1| < k_{\max} \equiv \left| \frac{2\omega_{\max}}{k_0(c - \bar{c})} \right|.$$

When $|k_1| = k_{max}$ there is no amplification, Im $\omega_1 = 0$.

An important particular case requiring special consideration is when $n = 0$. Geometrically this case corresponds to the inflexion of the axis of rotation. Such an instability can easily be recorded by experiment and it was observed only in [13, 14]. For $n = 0$, Eqs. (4.4) are considerably simplified and make it possible to compute $\omega_1$ at secondary intersection points. At principal intersection points $\omega_0 \neq 0$ and Eqs. (4.4) are not applicable since the condition $\omega_0 = 0$ was used in one of the intermediate steps. A separate consideration of the case $n = 0$, $\omega_0 = 0$ gives

$$\omega_1 = \pm \frac{3i}{8(2k_0^2 + 1)} \left[(3k_0^2 + 1)^2 - 16k_1^2(k_0^2 + 1)^3/k_0^2\right]^{1/2}.$$

The root with Im $\omega_1 > 0$ exists when

$$k_1^2 < k_{\max}^2 = \frac{k_0^2(3k_0^2 + 1)}{16(k_0^2 + 1)^2}. \tag{4.5}$$

When $k_1 = 0$, this root corresponds to the maximum

$$\omega_{\max} = \frac{3i}{8} \frac{3k_0^2 + 1}{2k_0^2 + 1}. \tag{4.6}$$

The first two columns in Table 2 give values of $-i\omega_{max}$ and $k_{max}$ for $n = 0$ at all intersection points present in the figure. It is seen that all intersection points indicate instability but for the secondary intersection points, the quantities $-i\omega_{max}$, $k_{max}$ are one to two orders of magnitude less than that of the principal intersection points. A similar situation also exists for $n = 1$, 2, and 3. Corresponding values of $i\omega_{max}$ are given in the remaining columns of Table 2. The consideration of negative $n$ does not give anything new since results for $n = n_0$ and $n = -n_0$ coincide to the same accuracy as the replacement of $\omega_0$ by $-\omega_0$. Conclusions for $|n| > 3$, are, apparently, similar.

Thus, there is a general conclusion on the dominating role of instabilities corresponding to principal intersection points of dispersion curves. When $n = 0$ these instabilities lead to inflection in the axis of rotation and are irrotational ($\omega_0 = 0$). Only this type of instability was observed so far in experiments [13, 14]. Instabilities with $n \neq 0$ lead to streamline distortion without a change in the axis of rotation and are rotational ($\omega_0 \approx n$). Their experimental record will be of interest. We emphasize here that the amplification rate for disturbances with $n = 0$ and $n \neq 0$ are practically identical.

Comparing the analytical results obtained with the earlier known results, we observe that the particular case $n = 0$ was studied in [13, 14] using the Galerkin method. Only the instability corresponding to, in our terminology, principal intersection points, was considered. Exact agreement of these results with [13, 14] is observed only for the coordinates of these points whereas, amplification rate (4.6) and the width of unstable zones (4.5) are somewhat different. Since the Galerkin method which involves additional assumptions was not used here, results of the present paper, in particular, establish the conclusions of [13, 14].

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